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Value of the Cosmological Constant in the Cosmological Relativity Theory

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It is shown that the cosmological relativity theory predicts the value $\Lambda = 1.934 \times 10^{-35}\mathrm{s}^{-2}$ for the cosmological constant. This value of Λ is in excellent agreement with the measurements recently obtained by the High-Z Supernova Team and the Supernova Cosmology Project.

1. INTRODUCTION

The problem of the cosmological constant and the vacuum energy associated with it is of high interest these days. There are many questions related to it at the quantum level, all of which are related to quantum gravity. Why there exists the critical mass density and why the cosmological constant has this value? Trying to answer these questions and others were recently the subject of many publications (Adler, 1997; Axenides *et al.*, 2000; Carroll *et al.*, 1992; Carroll, 2000; Cohn, 1998; Estrada and Masperi, 1998; Fujii, 2000; Garriga *et al.*, 2000; Garriga and Vilenkin, 2000; Goliath and Ellis, 1999; Guendelman and Kaganovich, 1998; Roos and Harun-or-Rashid, 1998; Rubakov and Tinyakov, 2000; Sahni and Starobinsky, 2000; Weinberg, 2000a,b; Witten, 2000; Zlatev *et al.*, 1999).

In this paper it is shown that the cosmological relativity theory (Behar and Carmeli, 2000) predicts the value $\Lambda = 1.934 \times 10^{-35}\mathrm{s}^{-2}$ for the cosmological constant. This value of Λ is in excellent agreement with the measurements recently obtained by the High-Z Supernova Team and the Supernova Cosmological Project (Garnavich *et al.*, 1998a,b; Perlmutter *et al.*, 1997, 1998, 1999; Riess *et al.*, 1998; Schmidt *et al.*, 1998).

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2. THE COSMOLOGICAL CONSTANT

In 1922 Friedmann solved the Einstein gravitational field equations and obtained nonstatic cosmological solutions presenting an expanding universe (Friedmann, 1922, 1924). Einstein, who thought at that time that the universe should be static and unchanged forever, suggested a modification to his original field equations by adding to them the so-called cosmological term that can stop the expansion. The field equations with the added term are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \tag{1}$$

where Λ is the cosmological constant, the value of which is supposed to be determined experimentally. In Eq. (1), $R_{\mu\nu}$ and R are the Ricci tensor and scalar, respectively, $\kappa = 8\pi G$, where G is Newton's constant, and the speed of light is taken as unity.

Soon after that Hubble (1927, 1936) found experimentally that the distant galaxies are receding from us, and the farther the galaxy the bigger its velocity as determined by its redshift.

After Hubble's discovery that the universe is expanding, the role of the cosmological constant to allow static homogeneous solutions to Einstein's equations in the presence of matter, was looked upon as unnecessary. For a long time the cosmological term was considered to be of no interest in cosmological physical problems.

3. THE FRIEDMANN UNIVERSE

For a homogeneous and isotropic universe with the line element (Landau and Lifshitz, 1979; Ohanian and Ruffini, 1994)

$$ds^2 = dt^2 - a^2(t)R_0^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right], \tag{2}$$

where k is the curvature parameter ($k = 1, 0, -1$) and $a(t) = R(t)/R_0$ is the scale factor, with the energy–momentum tensor

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \tag{3}$$

Einstein's equations (1) reduce to the two Friedmann equations

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2R_0^2}, \tag{4}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho + 3p) + \frac{\Lambda}{3}. \tag{5}$$

In Eqs. (4) and (5), H is Hubble's parameter, ρ is the mass density, and p is the pressure. These equations admit a static solution ($\dot{a} = 0$) with $k > 0$ and $\Lambda > 0$.

From the Friedmann equation (4) it then follows that for any value of the Hubble parameter H there exists a critical mass density $\rho_c = 3H_0^2/\kappa$ at which the spatial geometry is flat ($k = 0$). One usually measures the total mass density in terms of the critical density ρ_c by means of the density parameter $\Omega = \rho/\rho_c$.

In general, the mass density ρ includes contributions from various distinct components. From the point of view of cosmology, the relevant aspect of each component is how its contribution to the total energy density evolves as the universe expands. A positive Λ causes acceleration to the universe expansion, whereas a negative Λ and ordinary matter tend to decelerate it. Moreover, the relative contributions of the components to the energy density change with time. For $\Omega_\Lambda < 0$, the universe will always recollapse to a Big Crunch. For $\Omega_\Lambda > 0$, the universe will expand forever unless there is sufficient matter to cause recollapse before Ω_Λ becomes dynamically important. For $\Omega_\Lambda = 0$, we have the familiar situation in which $0 < \Omega_M \leq 1$ universes expand forever and $\Omega_M > 1$ universes recollapse. (For more details see Behar and Carmeli, 2000.)

4. THE SUPERNOVAE EXPERIMENTS

Recently two groups (the Supernova Cosmology Project and the High-Z Supernova Team) presented evidence that the expansion of the universe is accelerating (Garnavich *et al.*, 1998a,b; Perlmutter *et al.*, 1997, 1998, 1999; Riess *et al.*, 1998; Schmidt *et al.*, 1998). These teams have measured the distances to cosmological supernovae by using the fact that the intrinsic luminosity of Type Ia supernovae is closely correlated to their decline rate from maximum brightness, which can be independently measured. These measurements, combined with redshift data for the supernovae, led to the prediction of an accelerating universe. Both teams obtained

$$\Omega_M \approx 0.3, \qquad \Omega_\Lambda \approx 0.7, \tag{6}$$

and strongly ruled out the traditional $(\Omega_M, \Omega_\Lambda) = (1, 0)$ universe. This value of the density parameter Ω_Λ corresponds to a cosmological constant that is small but, nevertheless, nonzero and positive, that is,

$$\Lambda \approx 10^{-52}\mathrm{m}^{-2} \approx 10^{-35}\mathrm{s}^{-2}. \tag{7}$$

5. THE COSMOLOGICAL RELATIVITY THEORY

In Behar and Carmeli (2000), a four-dimensional cosmological relativity theory that unifies space and velocity was presented. The theory predicts that the universe accelerates and hence it is equivalent to having a positive value for Λ in it. As is well known, when a cosmological constant is added in the traditional work of Friedmann, the field equations obtained are highly complicated and no solutions have been obtained so far.

Cosmological relativity theory, on the other hand, yields exact solutions and describes the universe as having a three-phase evolution, with a decelerating expansion followed by a constant and an accelerating expansion, and it predicts that the universe is now in the latter phase. In the framework of this theory, the zero–zero component of Einstein's equations is written as (Behar and Carmeli, 2000)

$$R_0^0 - \frac{1}{2}\delta_0^0 R = \kappa\rho_{\rm eff} = \kappa(\rho - \rho_{\rm c}), \tag{8}$$

where $\rho_{\rm c} = 3/\kappa\tau^2 \approx 3H_0^2/\kappa$ is the critical mass density and τ is Hubble's time in the zero-gravity limit.

Comparing Eq. (8) with the zero–zero component of Eq. (1), one obtains the expression for the cosmological constant in cosmological relativity theory,

$$\Lambda = \kappa\rho_{\rm c} = 3/\tau^2 \approx 3H_0^2. \tag{9}$$

Assuming that Hubble's constant $H_0 = 70$ km/s-Mpc, then

$$\Lambda = 1.934 \times 10^{-35}{\rm s}^{-2}. \tag{10}$$

This result is in excellent agreement with the recent supernovae experimental results.

6. CONCLUSIONS

We have seen how the cosmological constant can be determined in a natural way without even adding it explicitly to the Einstein field equations. Rather, the introduction of the effective mass density $\rho_{\rm eff} = \rho - \rho_{\rm c}$ is enough to ensure that the universe expands in the same way using Einstein's field equations with a cosmological constant. But there is a big difference now: The theory determines the numerical value of the cosmological constant, and experiments confirm it.

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